PROBLEMS AND SOLUTIONS.

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PROBLEMS FOR SOLUTION.

ALGEBRA.

463. Proposed by H. O. HANSON, East Elmhurst, N. Y.

Find the sum of the series

$$\binom{2n}{0} + 2\binom{2n-1}{1} + 2^2\binom{2n-2}{2} + \cdots + 2^n\binom{n}{n},$$

where $\binom{n}{r}$ denotes the coefficient of x^r in the expansion of $(1+x)^n$.

464. Proposed by GEORGE Y. SOSNOW, Newark, New Jersey.

Find the general term and the sum of n terms of the series 1, 4, 15, 56, $\cdots$, where

$$U_n = 4U_{n-1} - U_{n-2}.$$

GEOMETRY.

495. Proposed by N. P. PANDYA, Sojitra, India.

A point P moves so that the quadrilateral $PBCD$ is half of a given quadrilateral $ABCD$. Find the locus of P.

496. Proposed by NATHAN ALTSHILLER, University of Colorado.

Find all the lines such that the pairs of tangent planes to a given sphere (ellipsoid) passing through them, shall be orthogonal.

CALCULUS.

413. Proposed by OSCAR S. ADAMS, U. S. Coast and Geodetic Survey, Washington, D. C.

Determine a function of x independent of b such that $\int_b^{b+1} f(x)dx = \frac{1}{b+1}$, the real part of b being positive.

414. Proposed by C. N. SCHMALL, New York City.

Among spherical triangles having the same base and equal altitudes, show that the isosceles triangle has the greatest vertical angle.

Show that this is also true for plane triangles.

MECHANICS.

330. Proposed by PAUL CAPRON, U. S. Naval Academy.

A barker's mill operates under a head of h feet; the linear speed of the orifice is u feet per second; the speed of the water relative to the orifice is v feet per second; and the coefficient of discharge is c, so that $v^2 = c^2(2gh + u^2)$. Given that the work done by the water on the mill is $u(v-u)/g$ foot-pounds per second per pound of water used, find the values of u and v such that the water-power may be most economically used, and find what part of the power is so used.

331. Proposed by CLIFFORD N. MILLS, Brookings, S. Dakota.

A cyclist is riding due west at a speed of 12 miles per hour, and the wind is at the time blowing from the southeast with a speed of 5½ miles per hour. If the cyclist carries a small flag, in what direction will this flag fly? At what speed would the cyclist need to ride if the flag is to fly due north?